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THE CORE BINARY FRACTIONS OF STAR CLUSTERS FROM REALISTIC SIMULATIONS

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ABSTRACT

We investigate the evolution of binary fractions in star clusters using *N*-body models of up to 100,000 stars. Primordial binary frequencies in these models range from 5% to 50%. Simulations are performed with the NBODY4 code and include a full mass spectrum of stars, stellar evolution, binary evolution, and the tidal field of the Galaxy. We find that the overall binary fraction of a cluster almost always remains close to the primordial value, except at late times when a cluster is near dissolution. A critical exception occurs in the central regions, where we observe a marked *increase* in binary fraction with time—a simulation starting with 100,000 stars and 5% binaries reached a core binary frequency as high as 40% at the end of the core-collapse phase (occurring at 16 Gyr with ~20,000 stars remaining). Binaries are destroyed in the core by a variety of processes as a cluster evolves, but the combination of mass segregation and creation of new binaries in exchange interactions produces the observed increase in relative number. We also find that binaries are cycled into and out of cluster cores in a manner that is analogous to convection in stars. For models of 100,000 stars we show that the evolution of the core radius up to the end of the initial phase of core collapse is not affected by the exact value of the primordial binary frequency (for frequencies of 10% or less). We discuss the ramifications of our results for the likely primordial binary content of globular clusters.

Subject headings: binaries: close — binaries: general — globular clusters: general — methods: *n*-body simulations — open clusters and associations: general — stellar dynamics

1. INTRODUCTION

The binary content of a globular cluster is important in determining the frequency and nature of cluster stellar exotica, as well as the dynamical evolution of the cluster. It has long been recognized that binary formation is inevitable in a self-gravitating system.[1] Indeed, the presence of binaries as a central energy source is vital to avoid complete core collapse (Goodman & Hut 1989). However, only more recently has it been realized that globular clusters must also have formed with a sizeable binary population (see Hut et al. 1992 for an early review). That globular clusters harbor a mixture of dynamically formed and primordial binaries can be used to understand observations of their stellar content, such as the diverse blue straggler population in 47 Tucanae (Mapelli et al. 2004).

Knowledge of the likely primordial binary fraction of globular clusters is essential as input to models of globular cluster evolution. It also provides a constraint on the cluster formation process. Considering that the presence of binaries in the cluster core has a pronounced effect on the core properties and cluster evolution (Hut 1996), knowledge of the central binary frequency is also important. Indications are that this is relatively small—of order 20% (e.g., Bellazzini et al. 2002) or less (e.g., Cool & Bolton 2002)—when compared to the frequencies of binaries observed in the solar neighborhood (Duquennoy & Mayor 1991) and open clusters such as M67 (Fan et al. 1996), which are of order 50%.

It would be particularly useful to take measurements of the current binary fraction in globular clusters—whether that be in the core or outer regions—and extrapolate backward to gain a reliable determination of the primordial binary content. However, processes involved in the intervening cluster evolution make this difficult. For example, binaries can be formed and destroyed in a variety of interactions between cluster members (Hurley & Shara 2002). Binaries will, on average, be more massive than single stars and thus are affected differently by mass segregation. Also, the escape rates of single stars and binaries will differ. Finally, the internal evolution of the components of binaries can also lead to the binaries' destruction.

Current simulation techniques have been designed to model these (and other) processes (Aarseth 2003) and have reached the level of sophistication required to produce realistic cluster models. In this way the link between primordial and current cluster binary populations can be investigated directly (e.g., Hurley et al. 2005; Ivanova et al. 2005). Aarseth (1996) conducted an *N*-body simulation starting with 10,000 stars and a 5% binary frequency where notably the stars were drawn from a realistic initial mass function (IMF), the cluster was subject to the tidal field of the Galaxy, and both stellar and binary evolution were modeled. This model cluster had a half-life of about 2 Gyr, at which point the core binary frequency had risen to 20% primarily owing to mass segregation. Thus, binaries were not preferentially depleted. In this case it was not necessary to include a large initial binary fraction in order to halt core collapse and yield a significant observed

[1] The 10-body gravitational calculations of von Hoerner (1960) are the earliest *N*-body calculations published. They were continued until the first binary formed, at which point the calculations were halted.

abundance in the central regions. The earlier work of McMillan & Hut (1994) reported N-body simulations of 2000 stars or less and binary frequencies in the range of 5%–20%. They included the Galactic tidal field but only considered point-mass dynamics. McMillan & Hut (1994) showed that there is a critical primordial binary frequency of 10%–15%, below which the binaries are destroyed before the cluster dissolves owing to the tidal field. Furthermore, they found that above this critical value there exists a minimum possible binary mass fraction for the cluster—this result could be used with observations of present-day binary frequency to place limits on the primordial frequency. We note that the McMillan & Hut (1994) simulations were restricted to equal-mass stars, and the binaries were a factor of 2 heavier than single stars—this could give misleading results when applied to real clusters.[2]

These N-body simulations were definitely in the open cluster regime. Dynamical processes that destroy (and equally may create) cluster binaries are density dependent. In addition, the central stellar density of a cluster is a function of the number, N, of cluster members. Thus, it is not clear that these prior results apply to globular cluster conditions. More recently, Ivanova et al. (2005) have conducted Monte Carlo simulations of clusters with up to 5×10^5 members and core number densities ranging from 10^3 to 10^6 stars pc^{-3}. They show that an initial binary frequency of 100% is required to produce a current core binary frequency of 10% for a globular cluster such as 47 Tuc. Depletion of binaries in the cluster core is found to be the result of stellar evolution processes as well as three- and four-body dynamical interactions. It is our intention in this paper to test these claims by using direct N-body simulations of star clusters with up to $N = 100{,}000$ members initially.

One aspect that will affect the evolution of the cluster binary population is the orbital parameters of the primordial binaries—in particular the initial ratio of *hard* to *soft* binaries. The boundary between these two regimes is determined by the mean kinetic energy of the cluster stars (with binaries represented by their center-of-mass motion), where hard binaries have a binding energy in excess of 2/3 of the mean kinetic energy (Hut et al. 1992). We note that a useful estimate for the boundary in terms of the binary orbital separation is given by twice the cluster half-mass radius divided by N. In three-body single-binary star interactions hard binaries tend to harden and provide kinetic heating for the cluster (Heggie 1975; Hut 1983). Soft binaries are less strongly bound (and thus, on average, are wider) and are efficiently destroyed in three- and four-body encounters. As noted by Hut et al. (1992) it is for this reason that soft binaries are not generally included in cluster models. A common misconception is that the omission of soft binaries is to aid the speed of simulation; however, it is binaries near the hard/soft boundary that provide the main threat to efficient simulation (Aarseth 2003). The omission is more a realization that soft binaries have little impact on the cluster dynamics or exotic star formation, and so the focus is on the more *meaningful* binaries, so to speak. Neglecting soft binaries has the capacity to alter binary fractions in the halo of a model cluster, as binary encounters tend to occur in or near the cluster core. For this reason we attempt to account for any omitted soft binary populations when making binary fraction comparisons.

Our simulation method and initial conditions are detailed in §§ 2 and 3. Results are given in § 4, followed by discussion in § 5. We briefly summarize our results in § 6.

2. MODELS

All simulations used in this work were performed using the NBODY4 code (Aarseth 1999) on GRAPE-6 boards (Makino 2002) located at the American Museum of Natural History. NBODY4 uses the fourth-order Hermite integration scheme and an individual time step algorithm to follow the orbits of cluster members and invokes regularization schemes to deal with the internal evolution of small-N subsystems (see Aarseth 2003 for details). Stellar and binary evolution of the cluster stars are performed in concert with the dynamical integration as described in Hurley et al. (2001).

The results of four extensive simulations (detailed below) form the data set for this paper. We make use of data from two simulations that have previously been reported in the literature—a simulation starting with 95,000 single stars and 5000 binaries (Shara & Hurley 2006) and a simulation starting with 12,000 single stars and 12,000 binaries (Hurley et al. 2005). The former contained 100,000 members at birth, if we count each binary as one object, and thus had a primordial binary frequency of 5%. We refer to this as the K100-5 simulation. After about 9 Gyr of evolution the cluster membership was reduced by half, and at an age of 15–16 Gyr the model cluster had reached the end of the main core-collapse phase (associated with a minimum in core radius, after which the size of the core stabilizes, in relative terms). Figure 1a shows the behavior of the core radius as the K100-5 model evolves. Also shown is the 10% Lagrangian radius—the radius that encloses the inner 10% of the cluster by mass. From Figure 1a we see that initially the inner regions of the cluster expand owing to stellar evolution mass loss before two-body effects take over and drive a prolonged period of contraction. When the cluster is about 12 half-mass relaxation times old (as denoted across the top of Fig. 1a) the core radius reaches a minimum of 0.17 pc and the main core-collapse phase is halted. The 10% Lagrangian radius at this point is 0.94 pc. The core density of the model begins at 10^2 stars pc^{-3} and increases to a maximum of 10^4 stars pc^{-3} just before termination of the model at 20 Gyr.

The core radius in Figure 1 is actually the density radius commonly used in N-body simulations (Casertano & Hut 1985). It is calculated from the density-weighted average of the distance of each star from the density center (Aarseth 2003). This definition, in combination with the effects of three-body interactions and the movement of binaries across the core boundary, allows for the small-scale fluctuations in core radius observed in Figure 1. Such fluctuations could be smoothed out (see Heggie et al. 2006, for example), but we have chosen not to do this. This N-body core radius is distinct from observational determinations of core radius calculated, for example, from the surface brightness profile (SBP) of a cluster. As discussed by Wilkinson et al. (2003) there is no general relation between the two quantities, but usually the N-body value is the lesser of the two. This is supported by an in-depth analysis of the core radius evolution of the K100-5 simulation, which will be presented in an upcoming paper (J. R. Hurley 2007, in preparation). Preliminary results show that the core radius obtained from the two-dimensional projected SBP of the K100-5 model agrees well with the N-body core radius for the first 7 Gyr of evolution, but is about twice as large by the time the model reaches 16 Gyr of age. Thus, the binary fraction within the 10% Lagrangian radius may often be a better number to compare with

[2] Binaries would naturally be twice as massive as single stars, on average, if binaries form by random pairings independent of the stellar IMF. In general, correlated masses are assumed (e.g., Kroupa 1995), although the exact situation is unclear—the recent survey of stars in the solar neighborhood and in young open clusters compiled by Halbwachs et al. (2003) shows a distribution of mass ratios, q, with a broad peak between 0.2 and 0.7, but also a sharp peak for $q > 0.8$.

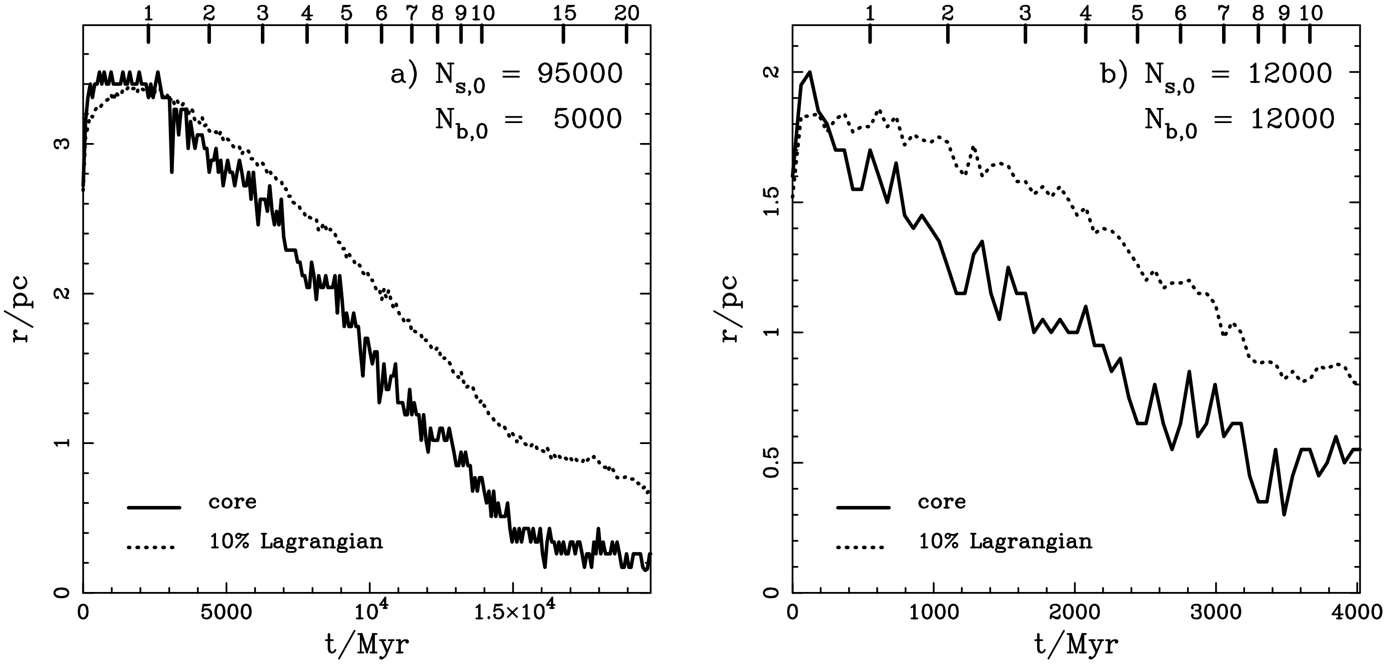

Fig. 1.—Evolution of the core radius (*solid lines*) and the radius containing the inner 10% of the cluster mass (*dotted lines*) for (*a*) the K100-5 simulation and (*b*) the K24-50 simulation. The numbers across the top show the number of half-mass relaxation times that have elapsed. Note that $N_{s,0}$ and $N_{b,0}$ refer to the number of single stars and binaries, respectively, in the starting model.

central binary fractions quoted for real clusters, and we give both this and the core binary fraction in our results.

The second model had a primordial binary frequency of 50% and was tailored to investigate the evolution and stellar populations of the old open cluster M67. It had 24,000 members at birth, and we refer to this as the K24-50 simulation. It had a half-life of about 2 Gyr, and after 4 Gyr of evolution only 2000 stars and binaries remained. The core density was about 10^2 stars pc^{-3}, on average, reaching a maximum of 350 stars pc^{-3} at 3480 Myr with a corresponding core radius of 0.3 pc. Figure 1*b* shows the evolution of the core and 10% Lagrangian radii for the K24-50 simulation.

To investigate the evolution of binary fractions across a range of star cluster models, we also make use of two simulations that have yet to be published. These are a simulation that started with 90,000 single stars and 10,000 binaries (K100-10) and a simulation that started with 40,000 single stars and 10,000 binaries (K50-20). In Table 1 we summarize the properties of the four simulations.

TABLE 1

Details of the Four *N*-Body Simulations Used in This Work

$N_{s,0}$ (1)	$N_{b,0}$ (2)	$\psi(a)$ (3)	$a_{\max}$ (4)	$f_{b,0}$ (5)	n_c (6)	$t_{1/2}$ (7)	Label (8)
95,000.............	5000	EFT30	100	0.05	10^2–10^4	8920	K100-5
90,000.............	10000	EFT30	100	0.10	100–500	8850	K100-10
40,000.............	10000	EFT30	50	0.20	10^3	5560	K50-20
12,000.............	12000	log *a*	50	0.50	100–350	2060	K24-50

Notes.—Cols. (1) and (2) show the number of single stars and binaries in the starting model. The distribution used to select the orbital separations of the primordial binaries is given in Col. (3), and this is followed by the maximum applied to the distribution (in AU). Col. (5) lists the primordial binary fraction, and in Col. (6) we show the typical stellar density in the core for the simulation (stars pc^{-3}). The half-life of the simulation (time in Myr for $N_s + N_b$ to drop to half the initial value) is given in Col. (7), and finally an identifying label is supplied for each simulation in Col. (8).

For each model the initial setup is as follows. Masses for the single stars are drawn from the IMF of Kroupa et al. (1993) between the mass limits of 0.1 and 50 $M_\odot$. Each binary mass is chosen from the IMF of Kroupa et al. (1991), as this had not been corrected for the effect of binaries, and the component masses are set by choosing a mass ratio from a uniform distribution. We assume that all stars are on the zero-age main sequence (ZAMS) when the simulation begins and that any residual gas from the star formation process has been removed. We use a Plummer density profile (Aarseth et al. 1974) and assume the stars and binaries are in virial equilibrium when assigning the initial positions and velocities. There is no primordial segregation by mass, binary properties, or any other discriminating factor in these models. Each cluster is subject to a standard Galactic tidal field—a circular orbit in the solar neighborhood. Stars are removed from the simulation when their distance from the density center exceeds twice that of the tidal radius of the cluster. The metallicity of the stars in the two simulations starting with 100,000 stars (K100-5 and K100-10) was set to be $Z = 0.001$, while both the K24-50 and K50-20 simulations were assigned solar metallicity ($Z = 0.02$).

3. BINARY PERIOD DISTRIBUTIONS

The orbital separations of the 5000 primordial binaries in the K100-5 simulation (Shara & Hurley 2006) were drawn from the lognormal distribution suggested by Eggleton et al. (1989) with a peak at 30 AU. This distribution is based on the properties of doubly bright visual binaries in the *Bright Star Catalogue* (Hoffleit 1983) and is in agreement with the survey data of Duquennoy & Mayor (1991) for binaries in the solar neighborhood—although the latter observations do not rule out a flat distribution. Orbital eccentricities of the primordial binaries were assumed to follow a thermal distribution (Heggie 1975). In the K100-5 model the initial separation distribution was capped at 100 AU. With a half-mass radius of 6.7 pc for the initial model the hard/soft binary boundary is at about 30 AU. Thus, the

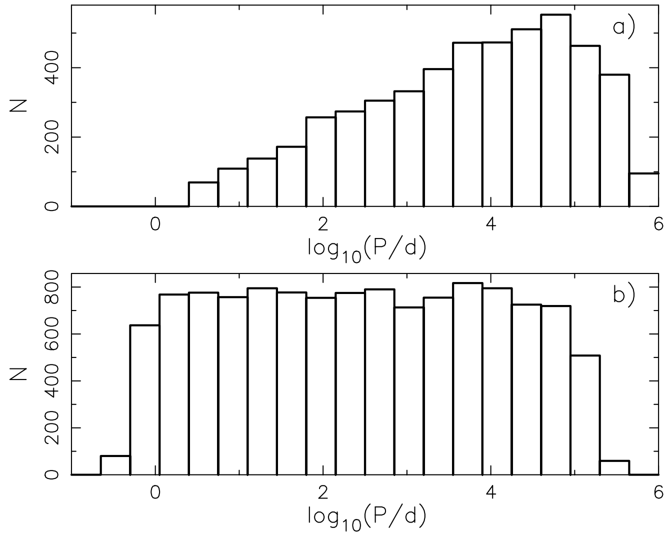

Fig. 2.—Period distribution of the primordial binary populations in (*a*) the K100-5 simulation (starting with 5000 binaries) and (*b*) the K24-50 simulation (starting with 12,000 binaries).

maximum of 100 AU excludes only the softest binaries from the distribution. Binaries with an initial pericenter distance less than 5 times the radius of the primary star were rejected in the setup of the model—for binaries closer than this it is assumed that interaction during the formation process and on the Hayashi track would lead to collision. Rather than enact such a collision we simply choose another set of binary parameters from the distributions. In this way the intended primordial binary fraction is preserved. The resulting period distribution of the K100-5 model is shown in Figure 2*a*. We see that the distribution is peaked at 10^5 days and does not extend beyond 10^6 days. The K100-10 simulation had the same binary setup as that of the K100-5 model. The K50-20 simulation also used the same Eggleton et al. (1989) distribution of orbital separations but with a cap at 50 AU.

Primordial binaries in the M67 (or K24-50) simulation of Hurley et al. (2005) have orbital separations drawn from a flat distribution of log *a* (Abt 1983). An upper cutoff of 50 AU was applied so that soft binaries were not included in the model—with a half-mass radius of 3.9 pc the hard/soft binary limit for the starting model was about 40 AU. For this model very close primordial orbits were also rejected. The corresponding period distribution for the primordial binaries in the K24-50 simulation is shown in Figure 2*b*. We note that a goal of the K24-50 simulation was to reproduce the relatively large number of blue stragglers observed in M67. For this purpose an Eggleton et al. (1989) separation distribution was ruled out, as it did not lead to enough blue straggler production from Case A mass transfer in close binaries. Uncorrelated masses of the component stars in binaries were also ruled out for the same reason (see Hurley et al. 2005 for details).

During this work we will make comparisons to the Monte Carlo models presented by Ivanova et al. (2005). In their study binary periods were chosen from a uniform distribution in log *P* between the limits of 0.1 and 10^7 days. Thus, they assumed a wider distribution of primordial binaries. If, for example, the Eggleton et al. (1989) distribution used in the K100-5 simulation was extended to include all periods up to 10^7 days, rather than being curtailed at 100 AU, the 5000 binaries that make up the distribution shown in Figure 2*a* would represent about 5/6 of the full population. So effectively there would be 1000 soft binaries that have been neglected, and the true primordial frequency would be 6%. One could then assume that these soft binaries were broken up at the very start of the simulation—although this may not be true for soft binaries residing in the less dense outer regions of the cluster. However, we note that there is no evidence that binary periods in star clusters should extend as far as 10^7 days (Meylan & Heggie 1997).

In terms of hard binaries one could argue, for the sake of semantics, that in comparison to a population drawn from a uniform distribution of periods extending from 0.1 days (without restriction) our initial distributions are undersampling the contribution of hard binaries. A key point here is that short-period binaries were not excluded from the primordial populations of our simulations by some ad hoc process. Instead, the distribution of orbital periods is dictated by using distributions borne from observations in combination with accounting for pre-main-sequence (pre-MS) evolution—before contracting along the Hayashi track the stellar radius of a pre-MS star can be a factor of 5 or more greater than on the ZAMS (Siess et al. 2000), and birth periods must allow for this (Kroupa 1995). Pre-MS evolution was not considered by Ivanova et al. (2005), although they did reject systems where one or both stars would initially fill their Roche lobes at pericenter—this was also assumed in our models.

4. RESULTS

In Figure 3 we show the evolution of the core binary fraction for the four *N*-body simulations introduced above. Also shown is the binary fraction within the 10% Lagrangian radius and the overall binary fraction of the model clusters.

Except at late times in the K24-50 model, when the cluster has lost more than 90% of its original mass and is nearing dissolution, we see that in each case the cluster binary fraction remains close to the primordial value. Focusing on the K100-5 simulation, Figure 4 shows the fractions of single stars and binaries (compared to their respective initial number) in the cluster. Following on from Figure 3*a* the fractions are similar at all times as expected. However, Figure 4 also shows the fractions of single stars and binaries that have escaped the cluster, and we see that from about 2 Gyr onward the fractional escape rate of single stars is greater than that of the binaries. At the end of the simulation (20 Gyr) the difference is 34%. This is offset somewhat by evolution processes (stellar and binary) that destroy binaries (see the dotted line in Fig. 4). These processes include binaries becoming unbound due to supernova mass loss and/or kicks (only relevant for the first 100 Myr of evolution) and mass transfer–induced mergers in close binaries. The remaining difference is balanced by the destruction of binaries in dynamical encounters, and this becomes more important as the cluster evolves. We note that even though the cluster binary fraction is relatively static as the cluster evolves, the characteristics of the binary population change markedly over time with hard binaries favored at late times.

Evident from Figure 3 is an overall trend for the core binary fraction to increase with time, irrespective of simulation type. For the core binary population of the K100-5 model we see that this rises from an initial 5% to as high as 40% around the time that the core-collapse phase is halted. After this time the core binary fraction becomes quite noisy owing to the small size of the core (see Fig. 1) and the small numbers of binaries and stars in the core. However, the value always remains greater than the initial value. We see also from Figure 3*a* that the binary frequency within the inner 10% Lagrangian radius rises to a maximum of 16% just prior to the end of the core-collapse phase.

It is important to note at this point that we are working with radii derived from spherical data, whereas observational

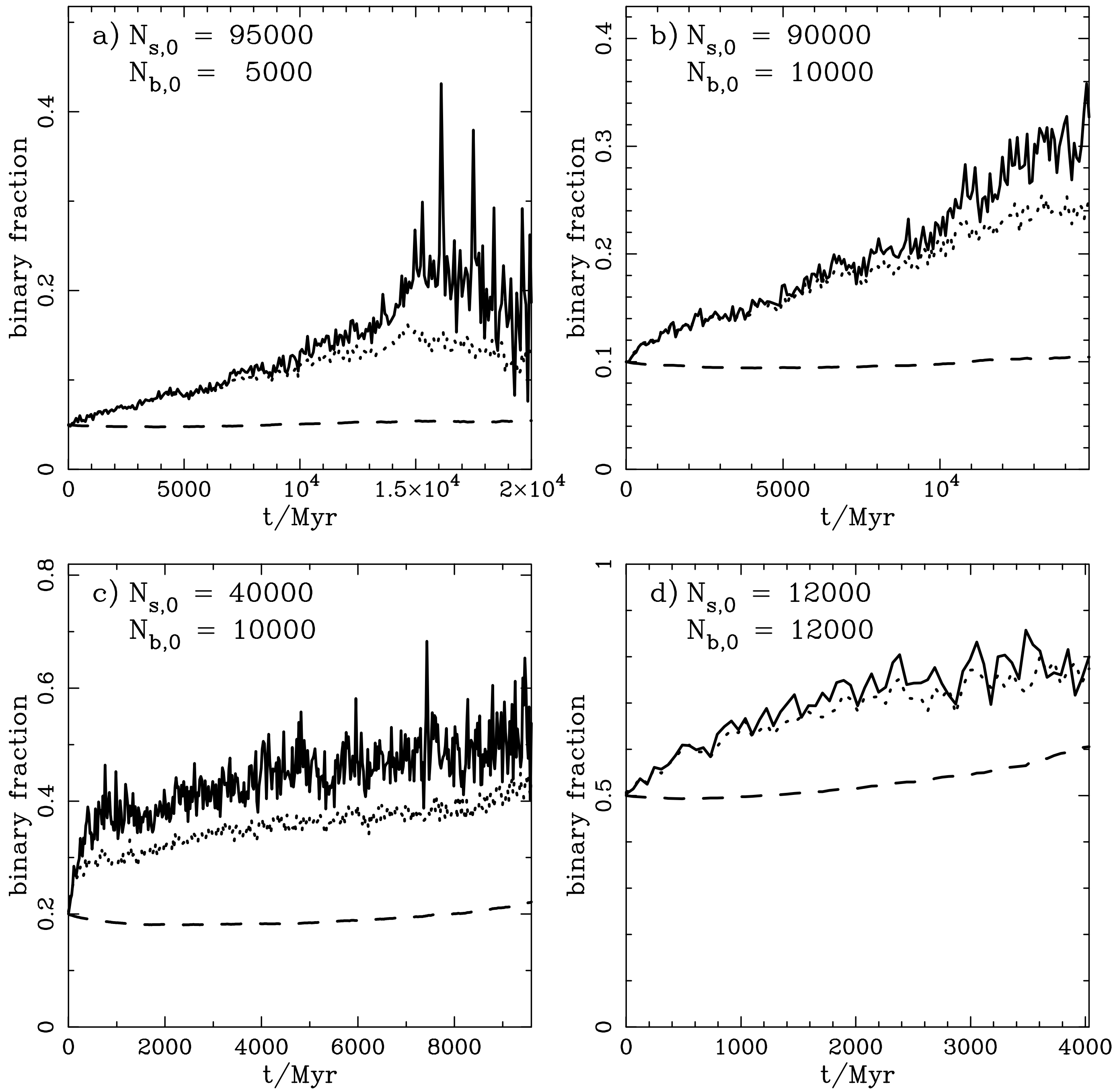

FIG. 3.—Evolution of the binary fraction in the core (*solid lines*), within the 10% Lagrangian radius (*dotted lines*), and for the entire cluster (*dashed lines*). Results are shown for the (*a*) K100-5, (*b*) K100-10, (*c*) K50-20, and (*d*) K24-50 simulations (see Table 1 for details).

determinations of binary fractions are based on two-dimensional projected data. With our models it is possible to test the effect of this discrepancy on our findings. If we calculate the 10% Lagrangian radius for model K100-5 from a two-dimensional projection, we find that the radius is reduced by about 20%–40% across the evolution (the choice of projection axis does not affect this result). This is consistent with the expectation suggested by Fleck et al. (2006). A similar relationship is reported by Baumgardt et al. (2005), in that the half-light radius (calculated from projected data) is approximately half the size of the half-mass radius (calculated from spherical data). However, the binary fraction within the projected 10% Lagrangian radius of our K100-5 model is almost indistinguishable from that of the result shown in Figure 3*a* (*dotted curve*).

We now aim to understand the processes underlying the evolution of the core binary fraction of star clusters, focusing again on the K100-5 simulation. Figure 5 shows the number of single stars and binaries in the core, relative to their total number in the cluster, as the cluster evolves. For the first 10 Gyr of evolution the ratio of binaries in the core to binaries in the cluster is fairly static—roughly one in 10 binaries is in the core. However, the ratio of single stars found in the core is decreasing sharply over the same time frame, and thus, single stars are being lost from the core at a greater rate than from the cluster in general (comparing

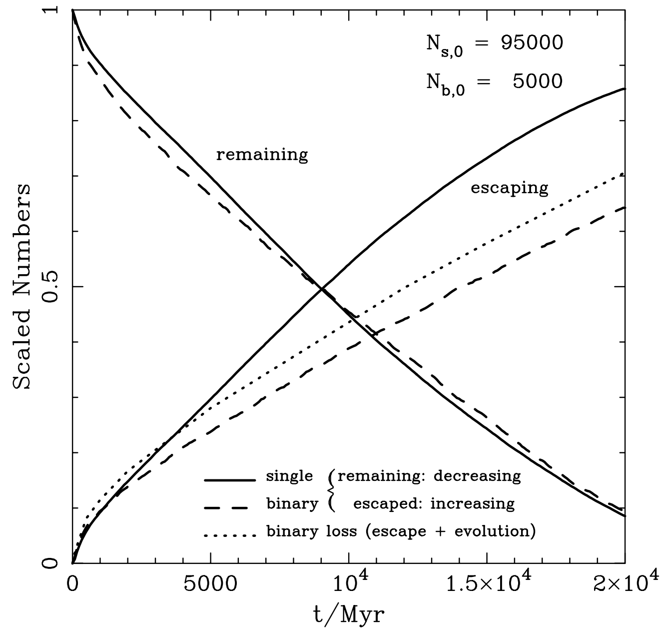

FIG. 4.—Fraction of single stars (*solid lines*) and binaries (*dashed lines*) remaining in the cluster as a function of time (lines decreasing from top left). Each population is scaled by the initial number of that population. Also shown are the fractions of single stars and binaries that have escaped from the cluster (lines increasing from bottom left). The dotted line is the combined fraction of binaries lost to escape and binary/stellar evolution processes. Results are for the K100-5 simulation.

Figs. 4 and 5). From 10 Gyr onward the ratio of binaries in the core also decreases. This corresponds to a period of increasing core density: prior to 10 Gyr the core density of stars hovers around the 10^2 stars pc^{-3} mark, but from 10 to 15 Gyr it increases by an order of magnitude. The binary fraction continues to rise in the core over this period, indicating that single stars continue to be lost from the core at a greater rate than binaries. We note that mass loss from stellar evolution is reduced considerably at this stage com-

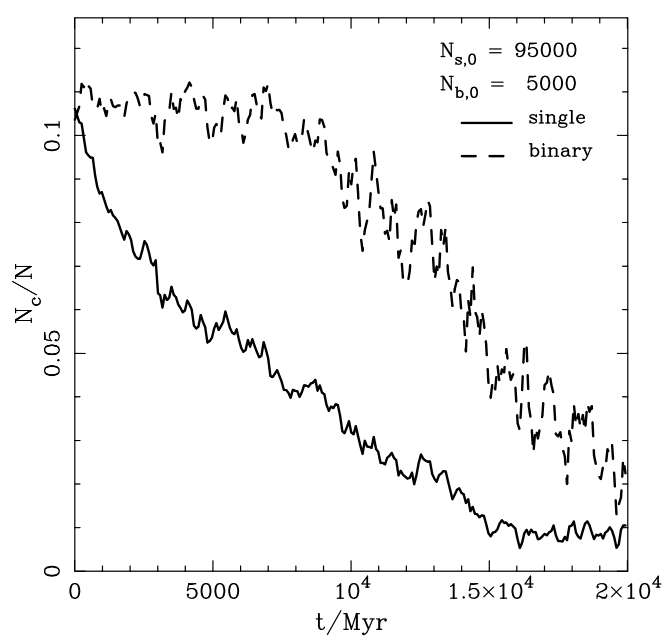

FIG. 5.—Number of single stars in the core as a fraction of the number of single stars in the cluster (*solid line*) and number of binaries in the core as a fraction of the number of binaries in the cluster (*dashed line*). Results are for the K100-5 simulation.

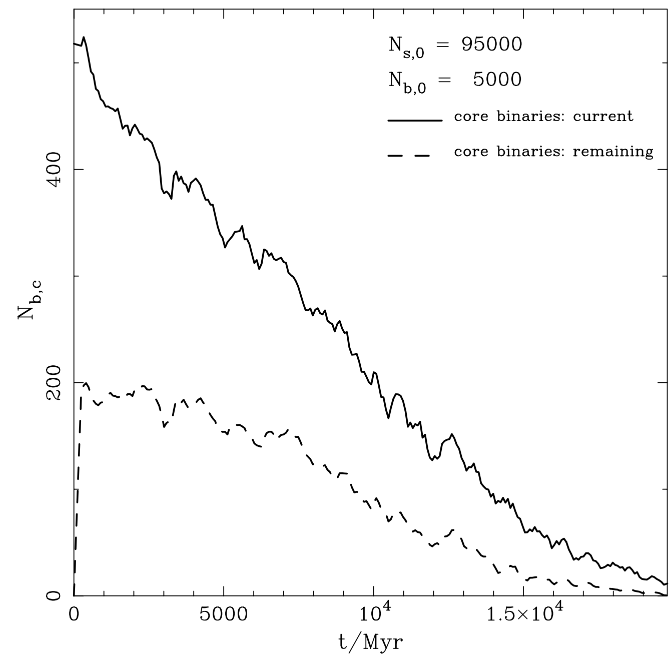

FIG. 6.—Number of core binaries as a function of time (*solid line*). Also shown at each time is the number of binaries that have remained in the core from the previous sampling (*dashed line*). Results are for the K100-5 simulation, and the data are sampled every 80 Myr.

pared to earlier in the cluster lifetime when more massive stars were present.

Figure 6 confirms that the *number* of binaries in the core is decreasing with time, even though the *binary fraction*, $f_{b,c}$, is increasing. We also see from this figure that at least half of the binaries in the core at any time were not present in the core the last time the population was sampled (this is done at intervals of 80 Myr). So the core binary population is by no means static, as many binaries are being created/destroyed, or moving in and out of the core, on the 80 Myr timescale. It is important to note for comparison that the relaxation time in the core is approximately 200 Myr initially and decreases to about 50 Myr at late times. *Individual binaries in cluster cores are both promiscuous and mobile-transient residents*.

In Figure 7 we examine the fraction of core binaries that were created in exchange interactions. These are short-lived three- and four-body gravitational encounters where a star is exchanged into an existing binary displacing one of the members of that binary (Heggie 1975). Thus, it is a process by which primordial binaries can be destroyed and replaced by new *dynamical*, or *exchange*, binaries. We see from Figure 7*a* that these nonprimordial binaries come to dominate the core population toward the end of the core-collapse phase in the K100-5 simulation. Figure 7*a* also shows that the double degenerate binary content increases steadily in the core with time and comprises about 30% of the core binaries subsequent to the completion of the core-collapse phase. In Figure 7*b* we see that the exchange binary content in the core of the K100-10 model does not reach the heights of the K100-5 model. Presumably this is a consequence of the lower core density of the K100-10 model. The fraction of double degenerate binaries is similar—any decrease in double degenerate production via dynamical means in the K100-10 model is compensated by the increased number of primordial binaries. The fraction of exchange binaries in the core of the K24-50 simulation (Fig. 7*d* is comparatively low, whereas the K50-20 simulation (Fig. 7*c* exhibits a much larger fraction. Clearly there is a positive correlation

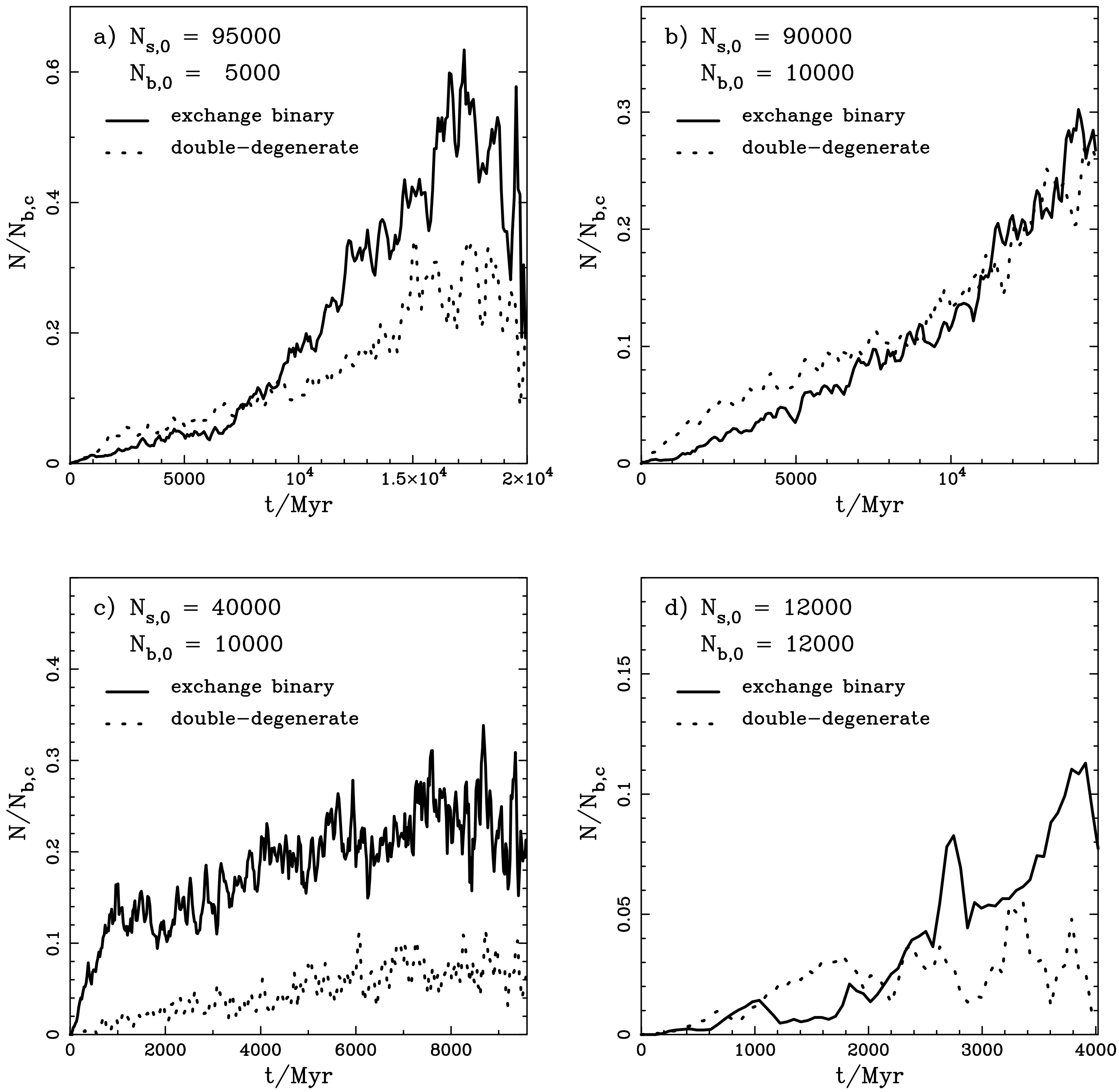

Fig. 7.—Fraction of binaries in the core that were created in an exchange interaction (*solid lines*) and fraction of core binaries that contain two degenerate stars (*dotted lines*). Results are shown for the (*a*) K100-5, (*b*) K100-10, (*c*) K50-20, and (*d*) K24-50 simulations (as described in Table 1).

between core density and the fraction of exchange binaries in the core.

Figure 8*a* shows the number of binaries created and destroyed in exchange interactions occurring in the core in intervals of 80 Myr. Also shown is the number of core binaries destroyed by all processes (exchanges, orbital perturbations, supernovae, and mergers) in each interval. The key point to note here is that, on average, exchange interactions are creating as many binaries as they are destroying. For the entire cluster there were 1024 binaries destroyed in exchange interactions during the simulation and 933 binaries created.

Figure 8*b* looks at the movement of binaries in and out of the core as the cluster evolves. Across each 80 Myr interval it shows the fraction of core binaries that move out of the core during the interval and the fraction of binaries that have moved into the core during the interval. We see that the inward and outward fluxes are equal. Also shown is the fraction of binaries entering the core that have previously been in the core—most binaries that leave the core eventually revisit it. We see a pattern where binaries move outward across the core boundary owing to recoil velocities from gravitational encounters, or as a result of the shrinking core. The core binary population is then replenished by binaries sinking inward owing to mass segregation effects. In the discussion below we refer to this pattern as *binary convection*. We note that binaries on radial orbits with a moderate to high eccentricity will also make an apparent contribution to this process.

An analysis of binary disruption for the K100-5 simulation is given in Figure 9 in terms of cumulative events. Exchange

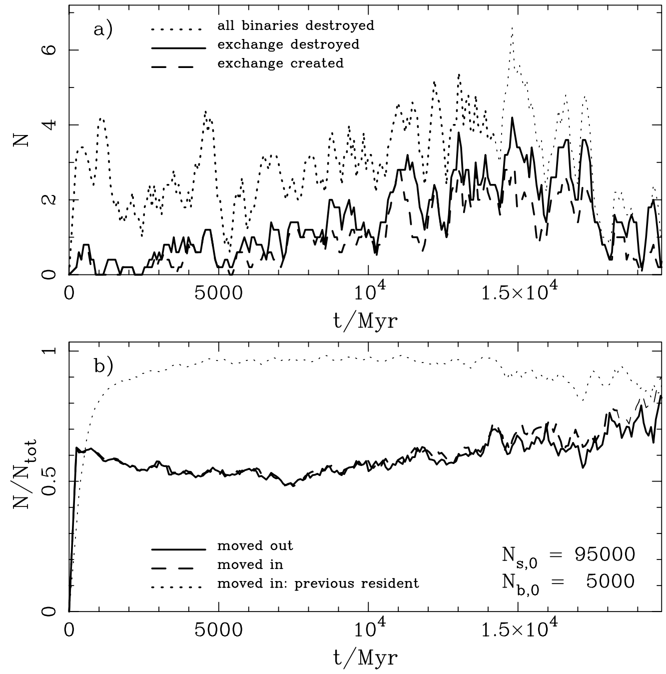

FIG. 8.—Statistics regarding core binaries across intervals of 80 Myr as the K100-5 model cluster evolves. Shown are (*a*) number of binaries destroyed in an exchange interaction occurring in the core (*solid line*), number of binaries created in exchange interactions in the core (*dashed line*), and the number of binaries destroyed by any means (*dotted line*); and (*b*) fraction of binaries that have moved out of the core but remained in the cluster (*solid line*: as a fraction of the number of binaries in the core at the start of the interval), number of binaries that have moved into the core (*dashed line*: as a fraction of the number of binaries in the core at the end of the interval), and the fraction of binaries entering the core that have previously resided in the core (*dotted line*). Note that the data have been moderately smoothed—over a width of 3 bins (or 240 Myr). Further smoothing would hide the naturally irregular behavior of the binary destruction/creation processes.

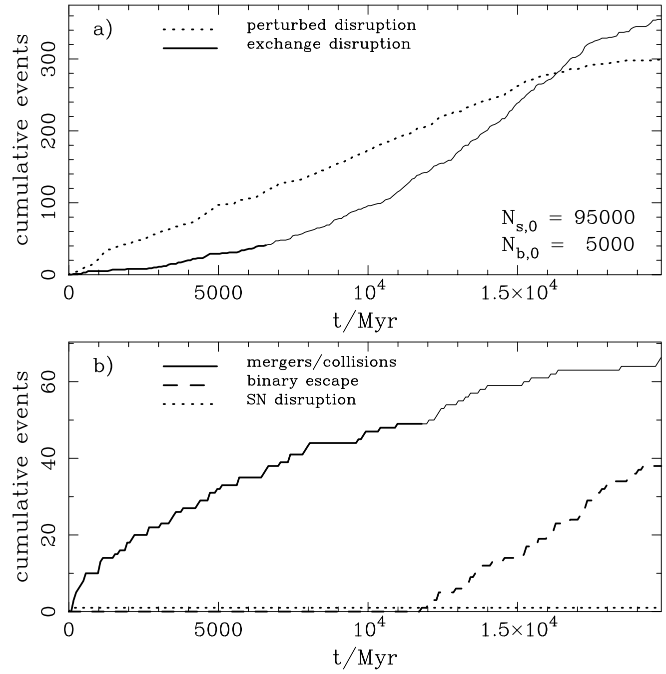

FIG. 9.—Cumulative numbers of events that led to the destruction of binaries in the core. Shown are (*a*) binaries broken up in exchange encounters (*solid line*) and binaries broken up owing to orbital perturbations (*dotted line*); and (*b*) binaries that were ejected from the core and escaped from the cluster (*dashed line*), binaries broken up as a result of supernovae explosions (*dotted line*), and binaries in which the stars merged (*solid line*)—this includes stellar evolution induced mergers and collisions at periastron in highly eccentric binaries. Results are for the K100-5 simulation.

interactions and orbital perturbations from nearby stars are by far the dominant causes of binary disruption, and these are shown in the top panel. We see that perturbation events are more likely at early times in the evolution, but as soft binaries disappear and the binary population becomes skewed toward hard binaries, exchange events eventually overtake perturbations as the major cause of disruption. However, there is an important distinction to make between these two types of events. Exchange interactions are counted as a disruption event in Figure 9*a* even if the event also leads to the creation of a new binary, and as we have seen in Figure 8*a* this is more than likely. On the other hand, if a binary is broken up owing to an orbital perturbation (also known as a flyby), there is no possibility of a replacement binary being created in the event.

The bottom panel of Figure 9 shows the number of binaries that were ejected from the core and escaped the cluster. There is a sharp correlation between the incidence of escape and the increase in core density after 10 Gyr. Even so, the total number of binaries lost owing to this process remains an order of magnitude less than either perturbation or exchange disruption. There is an initial burst of stellar/binary evolution induced mergers in short-period primordial binaries, followed by a gradual depletion of binaries owing to this process and collisions in highly eccentric binaries. The cluster had a total of 287 binaries that experienced either a merger or an internal collision, and 67 of these events occurred in the core. We also see from Figure 9*b* that supernova events do not make a meaningful contribution to depletion of the core binary population.

Figure 10 repeats Figure 9 for the K24-50 simulation. In this simulation mergers and collisions are the most likely cause of core binary loss. This is linked to the increased primordial binary fraction and decreased core density, compared to the K100-5 simulation. For similar reasons exchange disruption is more likely than perturbed disruption over the course of the evolution. In fact,

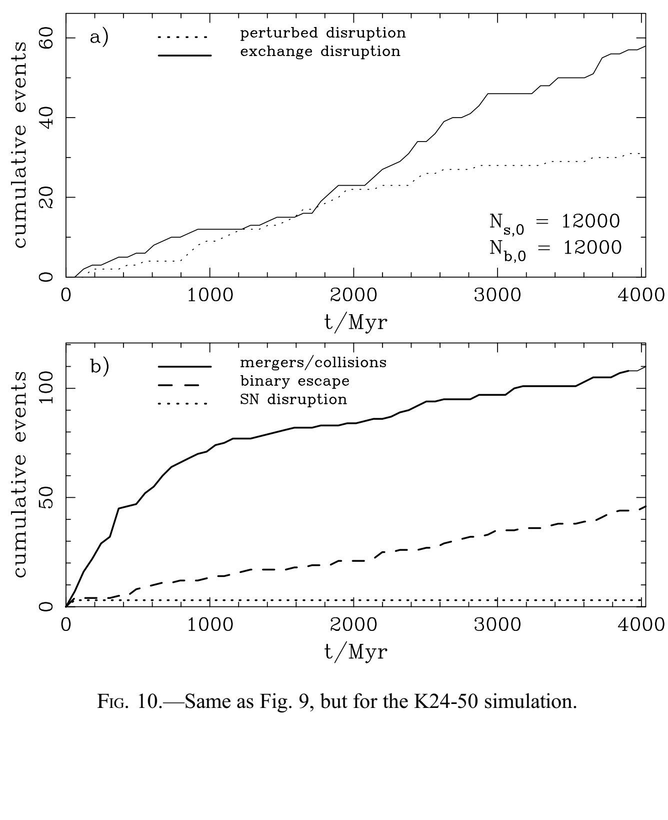

FIG. 10.—Same as Fig. 9, but for the K24-50 simulation.

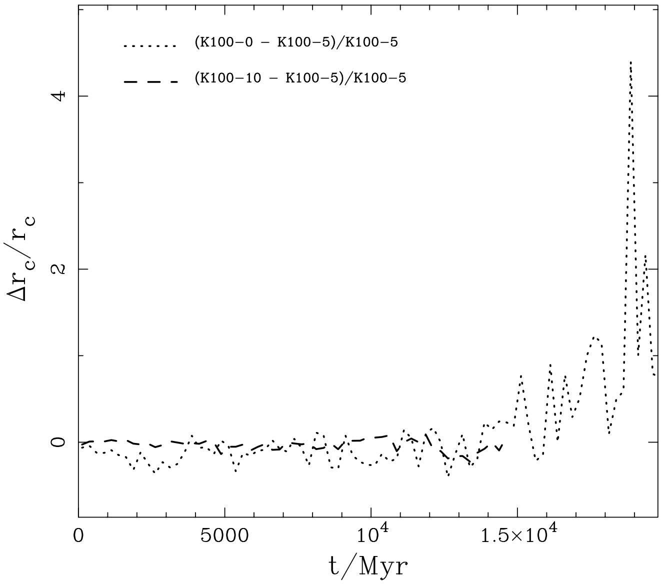

FIG. 11.—Comparison of core radius evolution for models starting with 100,000 stars. The K100-5 simulation is taken as a reference model, and shown are differences between the core radius of this model and models starting with 0% (*dotted line*) and 10% primordial binaries (K100-10: *dashed line*). The difference is scaled by the core radius of the K100-5 model. Note that for each simulation the core radius used is the average core radius in a 250 Myr interval.

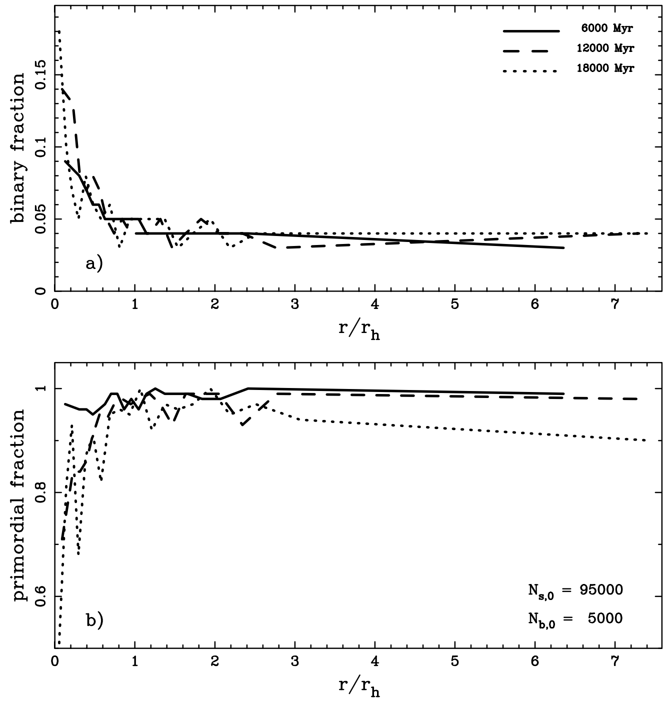

FIG. 12.—Binary data as a function of radial position for the K100-5 model. Shown at times of 6 Gyr (*solid lines*), 12 Gyr (*dashed lines*), and 18 Gyr (*dotted lines*) are (*a*) the distribution of binary fraction and (*b*) the fraction of binaries that are primordial. At each time there are 20 radial bins each containing the same mass, i.e., corresponding to Lagrangian radii incremented by 5%. Thus, the core is not resolved.

in this simulation even the loss of binaries from the core as a result of escape is greater than that from perturbed breakup. A key distinction between the K24-50 and K100-5 simulations is that in the K24-50 case the ratio of binary destruction to creation in exchanges is 3:1, whereas it was close to 1:1 for the K100-5 simulation.

The effect of a substantial primordial binary population on the evolution of open clusters has been documented in the past (McMillan et al. 1990, for example; see Meylan & Heggie 1997 for a review). The main results are that, in comparison to simulations without primordial binaries, the core-collapse phase of evolution is less dramatic and the cluster lifetime is reduced. Little has been done on this subject for globular clusters to date primarily because direct simulations have not been possible. However, our simulations starting with 100,000 stars can start to shed some light on the expected behavior. We see from Table 1 that increasing the primordial binary frequency from 5% (K100-5 simulation) to 10% (K100-10) does not reduce the cluster half-life significantly. In contrast, the K24-50 simulation with 50% binaries has a half-life of 2060 Myr, while a comparable simulation of 30,000 single stars with no primordial binaries has a half-life of 3600 Myr. As noted in Hurley & Shara (2003) the presence of a large number of primordial binaries in an open cluster leads to an enhanced rate of escaping stars via recoil velocity kicks obtained in three-body interactions. In comparison, the K100-5 and K100-10 clusters have deeper potential wells, and also the change in binary fraction between the two models is much less than for the open cluster example. So a sharp change in the escape rate is not to be expected.

Figure 11 shows that the core radius evolution of the K100-5 and K100-10 simulations is similar up to 15 Gyr (when the K100-10 simulation was stopped). We note, however, that the core density of the K100-10 model at this time is only half that of the K100-5 model. So the presence of additional primordial binaries has reduced the number density of stars in the core. Also in Figure 11 we compare the core radius evolution of a 100,000 star simulation with no primordial binaries (a K100-0 model). Here we see that the core radius evolution is slightly more irregular, but overall the evolution is once again similar up to 15 Gyr. After core collapse has been halted the situation is different, as the single star model experiences a fluctuating, and generally increasing, core radius, while the core radius of the K100-5 model remains approximately constant (see Fig. 1*a*). The K100-0 model has a greater core density than the K100-5 model at the end of the main core-collapse phase. The fluctuating core radius of the K100-0 model in the post–core collapse phase is indicative of the core bounce and subsequent oscillations expected for such a model—these phenomena are more pronounced for models without primordial binaries (see the related discussion in Heggie & Hut [2003] and Heggie et al. [2006]).

In Figure 12 we investigate the radial distribution of the K100-5 binary population at times of 6, 12, and 18 Gyr, i.e., before, during, and after the deep core-collapse phase. We see that outside of the half-mass radius the binary fraction is effectively constant with radius and changes little with time. The binary population in this region is also dominated by primordial binaries—exchange binaries are unlikely to be found outside of the half-mass radius. Within the half-mass radius the binary fraction rises sharply toward the center of the cluster and binaries become more centrally concentrated as the cluster evolves. Note that the inner radial bin corresponds to the inner 5% Lagrangian radius, so the core is not resolved in Figure 12.

5. DISCUSSION

Our *N*-body results clearly show that the core binary fraction of an evolved star cluster is expected to be greater than the primordial binary fraction. We see this behavior in each of the models presented and at all times in the evolution. The most striking case is our main model (K100-5), which started with 95,000 single stars and 5000 binaries and experienced a factor of 8 increase in the core binary fraction after 16 Gyr of evolution (coinciding with the end of the main core-collapse phase).

At face value our results appear to be in clear contradiction to the Monte Carlo results recently presented by Ivanova et al. (2005). Their main reference model has a primordial binary fraction $f_{b,0} = 1.0$ and a stellar density of $n_c = 10^5$ stars pc^{-3}. Processes such as exchange interactions, orbital perturbations, binary evolution, and mass segregation are included, and the model is reduced to $f_{b,c} = 0.095$ at 14 Gyr. Ivanova et al. (2005) then repeat the simulation with $f_{b,0} = 0.5$ and end up with $f_{b,c} = 0.07$, so, as they note, the relationship between primordial binary fraction and final core binary fraction is not linear. Coming at this from the other direction our models show a possible saturation effect as the primordial binary fraction increases. Looking back at Figures 3*a* and 3*b* we see that the core binary fraction of the K100-5 model at the 14 Gyr mark is 0.2 (up from $f_{b,0} = 0.05$), while it is 0.3 (up from $f_{b,0} = 0.1$) for the K100-10 model. So the simulation with the lower primordial binary fraction has experienced the greater relative increase in core binary content. Our K24-50 model, which started with $f_{b,0} = 0.5$, has $f_{b,c} = 0.8$ at a similar dynamical age, so the relative increase is less again. These results raise the possibility that decreasing $f_{b,0}$ below 0.5 in the Monte Carlo models may lead to conditions where $f_{b,c}$ can increase. It is interesting to note that the idealized models presented recently by Heggie et al. (2006) showed saturation effects in the core for initial binary frequencies greater than 10% and also recorded an increase in the core binary fraction with time.

Ivanova et al. (2005) also performed a model to compare with 47 Tuc. This was similar to their main reference model, although slightly more dense and with an increased velocity dispersion. The result for $f_{b,0} = 1.0$ was $f_{b,c} = 0.07$—this led to the conclusion that the primordial binary frequencies of globular clusters such as 47 Tuc must have been close to 100% to explain current observations. However, Ivanova et al. (2005) also ran the same simulation with $f_{b,0} = 0.75$, 0.5, and 0.25 and reported little or no variation in the final core binary fraction. It would seem safe to assume that repeating the simulation with $f_{b,0} = 0.1$ may give the same result or even an increase in binary fraction. This would act to remove any obvious discrepancies between the *N*-body and Monte Carlo results. We would certainly be interested in seeing the results of a Monte Carlo simulation conducted with $f_{b,0} = 0.1$ and a similar setup of the primordial binary population as used in this work—much easier than repeating a large *N*-body simulation with 100% binaries.

A major distinction between our *N*-body models and the Monte Carlo simulations mentioned above is that the stellar density is at least an order of magnitude greater in the latter. Fortunately, Ivanova et al. (2005) performed a simulation with $n_c = 10^3$ stars pc^{-3}, which facilitates a more direct comparison with our K50-20 model, which had a similar core density throughout the evolution. The K50-20 model experienced an almost factor of 2 increase in core binary fraction as it evolved from 0 to 8 Gyr. The comparable Monte Carlo model showed a reduction in core binary fraction of more than a factor of 2 over the same period. So there is an obvious deviation in behavior. Of course there is a large difference in the primordial binary fractions (0.2 compared to 1.0). The effect of this will be discussed further below. However, we note at this stage that the initial hard binary fraction in the Monte Carlo model was ~30% (N. Ivanova 2007, private communication), and this rose to 37%—so the hard binary fraction increases, and subsequently the models do show agreement at some level. Another consideration is the velocity dispersion, which is generally around 3–4 km s^{-1} for our models and was set to 10 km s^{-1} for most of the Monte Carlo models. However, Ivanova et al. (2005) did perform two models (D4 and M12) similar in all respects, except that $\sigma = 10$ km s^{-1} in one and 4.5 km s^{-1} in the other. There was no significant difference in the final core binary fractions of these models.

In § 3 we discussed that in the setup of our models we might be neglecting a fraction of soft binaries from the true primordial population. This results from imposing a maximum initial orbital separation and at most would cause the binary fraction to be underestimated by a few percent. Thus, we are confident that our choice of initial parameters for the binary populations in our models is not affecting the result that the core binary fraction increases as a cluster evolves. We also note that differences in the setup of primordial binaries between our simulations and those of Ivanova et al. (2005) make it difficult to directly compare quoted binary frequencies. For example, by not accounting for pre-MS stellar radii as we do, Ivanova et al. (2005) have a greater relative number of close binaries in their primordial populations. Such an excess would result in a greater number of evolution-induced binary mergers. If we were to adopt the period distribution and methods used by Ivanova et al. (2005) we would need to choose ~11,000 binaries in order to recover the 5000 in our K100-5 model at birth. This gives an effective primordial binary frequency of 11%, for the sake of comparison. The effective primordial binary frequency for the K24-50 simulation would be 80%. Adopting these values, in the worst case scenario, would still *not* lead us to conclude that the core binary fraction of an evolved cluster is decreased from the primordial value.

The comparable rates of binary disruption and creation owing to exchanges in our K100-5 simulation indicates that three-body interactions dominated over four-body interactions. This is because the most likely outcome of a binary-binary encounter is a binary and two single stars. So a binary is lost from the overall count. This is not the case for binary-single encounters where the most likely outcome is a binary and a single star, although the pairing of stars in the binary and/or the orbital parameters may have changed. By contrast, exchange interactions in the K24-50 simulation produced a binary disruption rate much higher than the binary creation rate. Here we had a much higher proportion of primordial binaries, and thus, binary-binary encounters were more likely. Thus, in terms of exchange interactions, increasing the primordial binary fraction can lead to a greater rate of binary destruction. This would certainly be expected to be true of models with comparable stellar densities. However, a competing effect comes from the fact that the central density is less for simulations with higher primordial binary fractions. We certainly see this when comparing our K100-5 and K100-10 models. The setup of these models was identical in all respects except for the change in primordial binary frequency from 5% to 10%. The models have similar half-lives, and we showed that the core radius evolution is also similar. So at any particular time in the evolution they are at a comparable dynamical age. But there is one clear difference—the model with twice as many primordial binaries has a central stellar density that is a factor of 2 less. This translates to a lower incidence of close stellar encounters, and as we saw from Figure 7 a greatly reduced fraction of exchange binaries in the core. Previous simulations, albeit with small-*N*, have indicated that the effects of primordial binaries saturate at some level (Wilkinson et al. 2003), so this is not necessarily a trend that we expect will continue as the primordial binary fraction is increased toward unity. However, it is certainly significant for clusters with frequencies of 10% or less.

Another point to note is that in a three-body exchange, not only is a binary not lost, but also a more massive single star is swapped for a less massive one, increasing the likelihood that the single star will be lost from the core via mass segregation. So the exchange process has indirectly increased the core binary fraction. The process of binary convection that became evident from Figure 8*b* is also related to mass segregation and acts to keep the core binary fraction healthy. Both single stars and binaries in the core are subject to velocity kicks from gravitational encounters. These kicks can remove an object from the core and even from the cluster entirely. For binaries this is less likely to occur primarily because they are, on average, more massive than single stars. Also, the average stellar mass decreases radially outward in an evolved cluster. So if a core binary suddenly finds itself outside of the core, it can be expected to be one of the more massive objects in its new local environment and thus to quickly sink back toward the core. We note that we found the movement of binaries inward and outward across the core boundary, as exhibited by Figures 6 and 8, to be quite striking.

Our K100-5 *N*-body simulation creates a realistic model of a moderate-sized globular cluster. It combines stellar and binary evolution with a self-consistent treatment of the cluster dynamics. It includes primordial binaries and accounts for the tidal field of the Galaxy. Thus, it provides us with a solid picture of how such a cluster evolves. Single stars escape from the cluster at a greater rate than binaries do—single stars are less massive, on average, so they are more likely to be tidally stripped after segregating to the outer regions of the cluster and also more likely to be ejected from the cluster in gravitational encounters. However, binaries are also lost from the cluster population owing to supernova disruption, evolution-induced mergers, and dynamical encounters. These effects balance, and the ratio of single stars to binaries is similar at all times in the evolution. As the cluster evolves, binaries sink toward the center and the binary fraction increases in the central regions. The core radius decreases as core collapse proceeds, and dynamical encounters become more prevalent. These encounters not only breakup binaries but also create new binaries. The cluster evolves to a state where primordial binaries dominate the binary population in the outer regions and nonprimordial binaries dominate toward the center.

In the center of the cluster soft binaries are broken up as a result of orbital perturbations from gravitational encounters. Binaries become involved in exchange interactions, primarily three-body, but these tend to create as many binaries as they destroy. Hard binaries are lost when the components merge as a result of close binary evolution or a collision at periastron. These are ongoing processes as the cluster evolves. At an age of 10 Gyr the rate of exchange interactions is greater than that of perturbed breakups and mergers. However, perturbed breakups are the dominant cause of binary loss. This is compared to the Monte Carlo model of Ivanova et al. (2005), which found that evolutionary mergers were the dominant event at the same age. We also find that after 10 Gyr, as the core density increases, binaries can be kicked out of the cluster directly from the core. Partly as a result of the combination of these processes the number of binaries in the core decreases as the cluster evolves. Also to blame is the movement of binaries outward across the core boundary owing to the decreasing size of the core and recoil velocities invoked in gravitational encounters. However, the movement of single stars outward across the core boundary is greater, and the net effect is an increase in the core binary fraction. This is also helped by binary convection where binaries that were previously resident in the core are cycled back in.

Noting that the typical membership of Galactic globular clusters exceeds 300,000 stars (Gnedin & Ostriker 1997, for example) we must ask the question—to what extent can we expect this behavior to extend to globular clusters in general? We can start with the ejection rate, $t_{\rm ej}$, of stars from an isolated cluster calculated by Hénon (1969), which gives $t_{\rm ej} \propto \ln(0.4N)\, t_{\rm rh}$ (Binney & Tremaine 1987). Here $t_{\rm rh}$ is the half-mass relaxation timescale, and we can relate this to behavior near the core of a cluster if we assume that core mass scales with total mass and that radii do not vary appreciably with cluster mass. This indicates that the relative rate of outward binary ejection and inward mass segregation (which occurs on a relaxation timescale) is only weakly dependent on the cluster mass. If we look in detail at the local relaxation timescale, this scales as

$$t_r \propto \frac{\sigma^3}{\rho \ln(0.4N)}$$

(Davies et al. 2004; as derived from Binney & Tremaine 1987), where σ is the velocity dispersion of the cluster stars and ρ is the mass density. We can take $\sigma \propto (M/r_h)^{1/2} \propto M_c^{1/2}$ and $\rho \propto M_c$, using the above assumptions, to show that $t_r \propto M_c^{1/2}/\ln(0.4N)$. Here M_c is the cluster core mass, M is the total cluster mass, and r_h is the half-mass radius. The timescale for a typical binary in the core of a globular cluster to have a close encounter with another star scales as

$$t_{\rm enc} \propto \frac{\sigma}{n}$$

(Davies et al. 2004), where n is the number density and $n \sim \rho$ if the average stellar mass is of order $M_\odot$, as it is in an evolved cluster core. This gives us $t_{\rm enc} \propto M_c^{-1/2}$. To escape the core, a binary must acquire a boost in energy of order $GM_c/2r_c$ (where G is the gravitational constant). So, assuming that the average energy imparted in an encounter does not vary strongly with mass, we have $t_{\rm ej} \propto M_c^{1/2}$. This rather simplified analysis returns Hénon's result and shows that as M (or N) increases there will be relatively less binary convection as both the ejection and relaxation timescales increase. However, the effect on the observed core binary fraction can be expected to be minimal.

We cannot definitively use our results to make predictions regarding globular clusters such as 47 Tuc because the central density in these clusters is at least an order of magnitude higher than that reached by our models. However, we note that our model with the highest core density showed the greatest increase in core binary fraction. Furthermore, we have considered a range of cluster types. *It does not appear, from our simulations, that an initial binary fraction anywhere near as high as 100% is required to give a core population of 20% or less at later times*. We also note that proper-motion cleaned color-magnitude diagrams recently presented for NGC 6397 (Richer et al. 2006) and M4 (Richer et al. 2004) show a distinct lack of binaries in regions outside of the cluster center—this cannot be reconciled with a large primordial binary population.

6. SUMMARY

We have presented a range of simulations typical of rich open clusters and moderate-sized globular clusters. In each case we find that the fraction of binaries in the core of a cluster does not decrease as the cluster evolves. In fact, the overriding trend is for an increase in core binary fraction from the primordial value. Thus, we do not agree with Ivanova et al. (2005) that the binary fraction in the core will be depleted in time. We also do not agree that models of globular cluster evolution need necessarily include large populations of primordial binaries.

Our simulations have shown that the binary population in the core of a cluster is continually being replenished by stars from outside the core, many of which were previously in the core. This is a process we have termed *binary convection*. We also find that the binary content of an evolved star cluster is dominated by exchange binaries provided that the stellar density is relatively high. This is true of our moderate-sized globular cluster models, and we expect it to be true in more massive clusters. We also show that increasing the primordial binary fraction does not necessarily lead to an increase in the final binary fraction—in fact, it gives more scope for binary depletion. A key and paradoxical result is that a final binary fraction that can be achieved by choosing a higher primordial binary fraction may also be replicated by choosing an initially lower binary fraction.

We find that the overall binary fraction of a cluster does not vary appreciably from the primordial value as a cluster evolves. This is a result of binary destruction being balanced by a greater rate of escape of single stars compared to binaries. We also find that the primordial binary frequency of a cluster is well preserved outside of the cluster half-mass radius. Therefore, observations of the current binary fraction in these regions is a good indicator of the primordial binary fraction, while determination of the core binary fraction provides an upper limit.

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REFERENCES

Aarseth, S. J. 1996, in Proc. IAU Symp. 174, Dynamical Evolution of Star Clusters: Confrontation of Theory and Observations, ed. P. Hut & J. Makiino (Dordrecht: Kluwer), 161
———. 1999, PASP, 111, 1333
———. 2003, Gravitational *N*-body Simulations: Tools and Algorithms (Cambridge: Cambridge Univ. Press)
Aarseth, S., Hénon, M., & Wielen, R. 1974, A&A, 37, 183
Abt, H. A. 1983, ARA&A, 21, 343
Baumgardt, H., Makino, J., & Hut, P. 2005, ApJ, 620, 238
Bellazzini, M., Fusi Pecci, F., Messineo, M., Monaco, L., & Rood, R. T. 2002, AJ, 123, 1509
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Casertano, S., & Hut, P. 1985, ApJ, 298, 80
Cool, A. M., & Bolton, A. S. 2002, in ASP Conf. Ser. 263, Stellar Collisions, Mergers, and Their Consequences, ed. M. M. Shara (San Francisco: ASP), 163
Davies, M. B., Piotto, G., & De Angeli, F. 2004, MNRAS, 349, 129
Duquennoy, A., & Mayor, M. 1991, A&A, 248, 485
Eggleton, P. P., Fitchett, M., & Tout, C. A. 1989, ApJ, 347, 998
Fan, X., et al. 1996, AJ, 112, 628
Fleck, J.-J., Boily, C. M., Lançon, A., & Deiters, S. 2006, MNRAS, 369, 1392
Gnedin, O. Y., & Ostriker, J. P. 1997, ApJ, 474, 223
Goodman, J., & Hut, P. 1989, Nature, 339, 40
Halbwachs, J. L., Mayor, M., Udry, S., & Arenou, F. 2003, A&A, 397, 159
Heggie, D. C. 1975, MNRAS, 173, 729
Heggie, D. C., & Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge: Cambridge Univ. Press)
Heggie, D. C., Trenti, M., & Hut, P. 2006, MNRAS, 368, 677
Hénon, M. 1969, A&A, 2, 151
Hoffleit, D. 1983, The Bright Star Catalogue (4th ed.; New Haven: Yale Univ. Obs.)
Hurley, J. R., Pols, O. R., Aarseth, S. J., & Tout, C. A. 2005, MNRAS, 363, 293
Hurley, J. R., & Shara, M. M. 2002, ApJ, 570, 184
———. 2003, ApJ, 589, 179
Hurley, J. R., Tout, C. A., Aarseth, S. J., & Pols, O. R. 2001, MNRAS, 323, 630
Hut, P. 1983, ApJ, 272, L29
———. 1996, in Proc. IAU Symp. 174, Dynamical Evolution of Star Clusters: Confrontation of Theory and Observations, ed. P. Hut & J. Makiino (Dordrecht: Kluwer), 121
Hut, P., et al. 1992, PASP, 104, 981
Ivanova, N., Belczynski, K., Fregeau, J. M., & Rasio, F. A. 2005, MNRAS, 358, 572
Kroupa, P. 1995, MNRAS, 277, 1507
Kroupa, P., Tout, C. A., & Gilmore, G. 1991, MNRAS, 251, 293
———. 1993, MNRAS, 262, 545
Makino, J. 2002, in ASP Conf. Ser. 263, Stellar Collisions, Mergers, and Their Consequences, ed. M. M. Shara (San Francisco: ASP), 389
Mapelli, M., Sigurdsson, S., Colpi, M., Ferraro, F. R., Possenti, A., Rood, R. T., Sills, A., & Beccari, G. 2004, ApJ, 605, L29
McMillan, S., & Hut, P. 1994, ApJ, 427, 793
McMillan, S., Hut, P., & Makino, J. 1990, ApJ, 362, 522
Meylan, G., & Heggie, D. C. 1997, A&A Rev., 8, 1
Richer, H. B., et al. 2004, AJ, 127, 2771
———. 2006, Science, 313, 936
Shara, M. M., & Hurley, J. R. 2006, ApJ, 646, 464
Siess, L., Dufour, E., & Forestini, M. 2000, A&A, 358, 593
von Hoerner, S. 1960, Z. Astrophys., 50, 184
Wilkinson, M. I., Hurley, J. R., Mackey, A. D., Gilmore, G. F., & Tout, C. A. 2003, MNRAS, 343, 1025